IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

Project Title:	**Mass-lumped high-order schemes for time-dependent problems**	
Project Number	**IMURA0828**	
Monash Main Supervisor (Name, Email Id, Phone)	Prof. Jerome Droniou jerome.droniou@monash.edu	*Full name, Email*
Monash Co-supervisor(s) (Name, Email Id, Phone)		
Monash Head of Dept/Centre (Name,Email)	Prof. Phil Hall, sci-maths-headofschool@monash.edu	*Full name, email*
Monash Department:	School of Mathematics, Faculty of Science	
Monash ADGR (Name,Email)	Prof. Peter Betts Peter.Betts@monash.edu	*Full name, email*
IITB Main Supervisor (Name, Email Id, Phone)	Prof. Neela Nataraj neela@math.iitb.ac.in	*Full name, Email*
IITB Co-supervisor(s) (Name, Email Id, Phone)		*Full name, Email*
IITB Head of Dept (Name, Email, Phone)	Prof. K. Suresh Kumar head.math@iitb.ac.in	*Full name, email*
IITB Department:	Department of Mathematics	

Research Clusters: / Research Themes:

	Highlight which of the Academy's CLUSTERS this project will address? *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)*		**Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)*
1	*Material Science/Engineering (including Nano, Metallurgy)*	*1*	*Advanced computational engineering, simulation and manufacture*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*	2	*Infrastructure Engineering*
3	*Math, CFD, Modelling, Manufacturing*	3	*Clean Energy*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*	4	*Water*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*	5	*Nanotechnology*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*	6	*Biotechnology and Stem Cell Research*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*	7	*Humanities and social sciences*
8	*HSS, Design, Management*	8	*Design*

The research problem

Define the problem

Many real-world models, especially those represented by Partial Differential Equations, involve non-linearities. These models are often too complex to be exactly solved, and numerical approximations are the only way to gather quantitative information on their solutions. In the context of numerical methods, it is well known that, for a given complexity of the method (cost to run simulations), a better accuracy is attained by using high-order schemes, based for example on local polynomials of order larger than one.

However, when applying standard high-order methods to certain types of non-linearities, one obtains a scheme whose accuracy cannot be ensured – techniques used to establish *a priori* error bounds fail. In these situations, a “mass-lumping” process has to be applied to transform the scheme and make it amenable, despite the severe non-linearities of the model, to a rigorous convergence analysis (unconditional stability, error estimates). This process can be understood in one of the three ways (an algebraic transformation of the scheme’s matrix, a particular choice of quadrature rules, or the usage of piecewise-constant functions in the design of the scheme); all of them, however, have the apparent effect of reducing the approximation to the lowest possible order, thus preventing the design of high-order schemes for such models.

Project aims

Define the aims of the project

The purpose of this project is to develop mass-lumped high-order numerical methods for non-linear PDEs, and to rigorously and numerically establish their accuracy. The models we will consider include in particular the Stefan model of melting material and the porous medium equation (describing the flow of a gas in a porous medium); both of them are degenerate parabolic problems with the kind of non-linearities that prevent classical high-order methods from being used for their approximation. An initial analysis has been conducted by A/Prof. Droniou and a collaborator on a stationnary version of these equations, and for schemes (Finite Element and Discontinuous Galerkin) based on triangular meshes. We aim at extending this approach to time-dependent models, non-triangular meshes, and other high-order methods such as the recent Hybrid High-Order scheme.
The accuracy of the designed methods will be assessed through a rigorous mathematical analysis, as well as thorough numerical tests.

Expected outcomes

Highlight the expected outcomes of the project

- Novel approach to mass-lumping.
- New high-order numerical methods for degenerate parabolic problems representing real-world models
- Rigorous numerical analysis of these schemes
- Numerical tests to compare the practical order of convergence in situations where the solution is not regular.

How will the project address the Goals of the above Themes?

Describe how the project will address the goals of one or more of the 6 Themes listed above.

This project addresses the theme *Advanced computational engineering, simulation and manufacture.* The

Stefan model and porous medium equations are both used to describe real-world models, whose complete understanding of solutions require the usage of numerical methods. By designing high-order schemes for these models, we ensure that our approximation will have a high accuracy and thus that the qualitative behaviour highlighted by our tests can be trusted, and that the schemes are thus usable in practical engineering situations. This trust will be reinforced by the rigorous mathematical analysis conducted on the designed methods.

Capabilities and Degrees Required

List the ideal set of capabilities that a student should have for this project. Feel free to be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.

- Basic theory of elliptic and parabolic partial differential equations
- Functional analysis
- Linear algebra
- Knowledge of classical Finite Element Methods
- Basic coding skills and interest

Potential Collaborators

Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.

Prof. Amiya K Pani, IIT Bombay
Prof. Harsha Hutridurga, IIT Bombay
Prof. Santiago Badia, Monash university (to arrive in July 2019)
A/Prof. Ricardo Ruiz-Baier, Monash University (to arrive in January 2020).

Select up to **(4)** keywords from the Academy's approved keyword list **(available at http://www.iitbmonash.org/becoming-a-research-supervisor/)** relating to this project to make it easier for the students to apply.

Computational Fluid Dynamics and Mechanics
Maths
Computer Simulation