IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

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| **Project Title:** | **Domain reconstruction in Electrical Impedance Tomography problems** | |
| **Project Number** | **IMURA0862** | |
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| **IITB Department**: | Mathematics | |

## Research Clusters: / Research Themes:

| **Highlight which of the Academy's CLUSTERS this project will address?** *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)* | | **Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)* | |
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| 1 | *Material Science/Engineering (including Nano, Metallurgy)* | *1* | *Advanced computational engineering, simulation and manufacture* |
| 2 | *Energy, Green Chem, Chemistry, Catalysis, Reaction Eng* | 2 | *Infrastructure Engineering* |
| 3 | *Math, CFD, Modelling, Manufacturing* | 3 | *Clean Energy* |
| 4 | *CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control* | 4 | *Water* |
| 5 | *Earth Sciences and Civil Engineering (Geo, Water, Climate)* | 5 | *Nanotechnology* |
| 6 | *Bio, Stem Cells, Bio Chem, Pharma, Food* | 6 | *Biotechnology and Stem Cell Research* |
| 7 | *Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng* | 7 | *Humanities and social sciences* |
| 8 | *HSS, Design, Management* | 8 | Design |

## The research problem

Electrical impedance tomography is a non-invasive imaging technique which has a great potential for complementing computerised tomography. Unfortunately, the associated mathematical problem of reconstructing the conductivity inside the body from measurements of the electrostatic potential of the skin, which is called the Calderon problem, is nonlinear and strongly ill-posed, so the desired images can only be computed with great difficulty.

This problem can partly be avoided by computing the convex source support, which is a convex body that approximates the subset of the body where the conductivity differs from the conductivity of healthy tissue, and not the conductivity itself. This idea has been introduced in [3] in the context of scattering, and it was transferred to electrical impedance tomography in [2].

A technique for the numerical computation of the convex source support as the solution of a set optimisation problem was proposed in [1]. Extending the approach to the situation where only a single measurement is available and the shape of the body is unknown is subject of current investigation. First results have been obtained, and a proof-of-principle implementation underpins the usefulness of the approach, but several interesting theoretical and computational details remain open. Moreover, numerical analysis aspects of the impedance tomography problem are an interesting area to focus on.

[1] Bastian Harrach and Janosch Rieger, A Set Optimization Technique for Domain Reconstruction from Single-Measurement Electrical Impedance Tomography Data, 2017 MATRIX Annals, 37-49, (2019).

[2] Martin Hanke, Nuutti Hyvönen, and Stefanie Reusswig
Convex Source Support and Its Application to Electric Impedance Tomography.
SIAM J. Imaging Sci., 1(4), 364–378, (2008).

[3] Kusiak, S, Sylvester, J.: The scattering support. Commun. Pure Appl. Math. 56(11), 1525–1548 (2003)

## Project aims

The student is supposed to work on some of the following objectives.

(a) Enhance the theoretical understanding of the set optimisation problem: local minima, error estimates under discretisation, and so on.
(b) Enhance the numerical analysis literature of inverse problems.
(c) Extend the current prototype algorithm from [1] from the space of rectangles to more general classes of convex sets. The main difficulty is the evaluation of the nonlinear state constraint, which involves the integration of a highly oscillatory function over a given set.
(d) Extend the prototype algorithm from 2d to 3d in theory and numerical implementation.
(e) Investigate the impact of multiple measurements on the stability of the reconstruction of the inhomogeneity, again in theory and numerical implementation.
(f) Explore alternative approaches such as monotonicity techniques in the set optimisation context.
(g) Implement optimisation software which is adapted to this particular set optimisation problem.

## Expected outcomes

1) novel algorithms to better describe the location of abnormal tissue through tomography
2) numerical analysis of these algorithms,
3) implementation and tests,
4) publications in high-quality journals.

## How will the project address the Goals of the above Themes?

Tomography is an essential non-invasive tool in medicine. Designing more robust and more accurate algorithms to analyse the results of a tomography scan is at the core of manufacturing software and apparatus to be used in practical medical situations. Since the currently available mathematical methods only allow a treatment of very basic applications, we will enhance the state of the art by exploring alternatives. In detail,

- tasks (a)-(d) and (f) contribute to the development of novel algorithms to better describe the location of abnormal tissue through tomography,
- tasks (a)-(f) will improve the understanding of the numerical analysis of these algorithms,
- task (g) will provide implementations and tests of the numerical methods, and
- the project in its entirety will lead to publications in high-quality journals.

## Capabilities and Degrees Required

- Masters Background in Analysis (Real, Complex, Functional Numerical), PDE, Linear Algebra, Optimization, Modern PDE, Scientific Computing (Preferable)

- Masters/M.Sc. in Mathematics

- Background in MATLAB, C++

## Potential Collaborators

*Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.*

1. K. Suresh Kumar
2. Andreas Ernst

Select up to **(4)** keywords from the Academy's approved keyword list **(available at http://www.iitbmonash.org/becoming-a-research-supervisor/)** relating to this project to make it easier for the students to apply.

Optimisation algorithms
Mathematics
BioScience, Bio Medical Engineering
Modelling and Simulation