IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

Project Title:	**Domain reconstruction in Electrical Impedance Tomography problems**	
Project Number	**IMURA0940**	
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Research Clusters: | Research Themes:

Highlight which of the Academy's CLUSTERS this project will address? *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)*		**Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)*	
1	*Material Science/Engineering (including Nano, Metallurgy)*	***1***	*Advanced computational engineering, simulation and manufacture*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*	2	*Infrastructure Engineering*
3	*Math, CFD, Modelling, Manufacturing*	3	*Clean Energy*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*	4	*Water*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*	5	*Nanotechnology*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*	6	*Biotechnology and Stem Cell Research*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*	7	*Humanities and social sciences*
8	*HSS, Design, Management*	8	Design

The research problem

The aim of this project is to explore and expand the numerical analysis of the impedance tomography problem, which, in mathematical terms, translates into identifying properties of the coefficients of an elliptic partial differential equation using certain measurement of the solution.

Electrical impedance tomography is an imaging technique, which has the potential to complement computerised tomography in applications such as screening and monitoring. While the tomograph is a rather simple device, the mathematical problem of reconstructing the exact interior structure of the body from measurements is very difficult, because it is nonlinear and ill-posed. These difficulties can partly be avoided by focussing on a simpler question: Can we detect some anomaly in the data, which could be caused by an unhealthy structure such as a tumour, and if so, where is this structure roughly located?

The convex source support is the smallest convex set that contains all anomalies within the imaged body. This concept has been investigated in [1] and [3]. In a sense, computing the convex source support answers the question raised above: If this set is empty, then there is no anomaly. If there is an anomaly, then the convex source support provides at least some information on its location. One technique for the numerical computation of the convex source support as the solution of a set optimisation problem was proposed in [2] under the unrealistic assumption that the exact shape of the imaged body is known precisely. First results have been obtained in the situation where the shape of the imaged body is unknown, but several interesting theoretical and computational details remain open.

The project will further extend this research and address the mathematical questions revolving around this PDE model and its numerical approximation. The research approach will involve a blend of partial differential equations, functional analysis, real and complex analysis, and basic topology.

[1] M. Hanke, N. Hyvönen, and S. Reusswig. *Convex source support and its application to electric impedance tomography.* SIAM J. Imaging Sci., 1(4), 364-378, (2008).

[2] B. Harrach and J. Rieger. *A set optimization technique for domain reconstruction from single-measurement electrical impedance tomography data.* 2017 MATRIX Annals, 37-49, (2019).

[3] S. Kusiak and J. Sylvester. *The scattering support.* Commun. Pure Appl. Math. 56(11), 1525-1548 (2003).

Project aims

The student is supposed to work on some of the following objectives.

(a) Enhance the theoretical understanding of the set optimisation problem: local minima, error estimates under discretisation, etc.
(b) Enhance the numerical analysis literature of inverse problems.
(c) Extend the current prototype algorithm from [2] from the space of rectangles to more general classes of convex sets.
(d) Extend the prototype algorithm from 2d to 3d in theory and numerical implementation.
(e) Investigate the impact of multiple measurements on the stability of the reconstruction of the inhomogeneity, again in theory and numerical implementation.
(f) Explore alternative approaches such as monotonicity techniques in the set optimisation context.
(g) Implement optimisation software, which is adapted to this particular set optimisation problem.

Expected outcomes

(1) novel algorithms to better describe the location of unhealthy tissue through tomography
(2) numerical analysis of these algorithms
(3) implementation and tests
(4) publications in high-quality journals

How will the project address the Goals of the above Themes?

Tomography is an essential non-invasive tool in medicine. Designing more robust and more accurate algorithms to analyse the results of a tomography scan is at the core of manufacturing software and apparatus to be used in practical medical situations. Since the currently available mathematical methods only allow a treatment of very basic applications, we will enhance the state of the art by exploring alternatives. In detail,

- tasks (a)-(d) and (f) contribute to the development of novel algorithms to better describe the location of abnormal tissue through tomography,
- tasks (a)-(f) will improve the understanding of the numerical analysis of these algorithms,
- task (g) will provide implementations and tests of the numerical methods, and
- the project in its entirety will lead to publications in high-quality journals.

Potential RPC members from IITB and Monash

(1) Harsha Hutridurga, Department of Mathematics, IIT Bombay
(2) Santiago Badia, Monash University Melbourne

Capabilities and Degrees Required

Masters Background in Analysis (Real, Complex, Functional, Numerical), PDE, Linear Algebra, Optimization, Modern PDE, Scientific Computing (Preferable)

Background in MATLAB, C++

Necessary Courses

Modern PDE
Analysis
Numerical Analysis

Potential Collaborators

K. Suresh Kumar
Andreas Ernst

Keywords

Mathematics
Data Science, optimisation, algorithms
Modelling and Simulation
Bioscience, Bio-Medical Engineering