

An Indian-Australian research partnership

**Project Title:** Distributed Algorithms for Mixed Integer Convex Optimization

**Project Number** IMURA0948

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**IITB Department:** Industrial Engineering and Operations Research

**Research Clusters:**

**Research Themes:**

<b>Highlight which of the Academy's CLUSTERS this project will address?</b> <i>(Please nominate JUST <b>one</b>. For more information, see <a href="http://www.iitbmonash.org">www.iitbmonash.org</a>)</i>	<b>Highlight which of the Academy's Theme(s) this project will address?</b> <i>(Feel free to nominate more than one. For more information, see <a href="http://www.iitbmonash.org">www.iitbmonash.org</a>)</i>																																
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## The research problem

### *Define the problem*

Optimization problems that involve a mixture of variables that represent discrete quantities (such as choices among a fixed number of options) and variables representing continuous quantities (such as route lengths or dollar amounts), are prevalent in a range of areas such as engineering design, logistics, computational finance, and machine learning. The need to design and analyse more complex systems, and gain insight from larger and larger datasets, requires reliable and scalable algorithms for such mixed-integer optimization problems. Nevertheless, it remains computationally challenging to solve large-scale instances of these problems, principally because of the presence of discrete variables.

More often than not, these optimization problems can be deconstructed into sub-structures which are well understood. Specifically, often we have mixed-integer optimization problems where optimizing over a subset of constraints is relatively easier, leading to the possibility of decomposition-based algorithms. This approach has been very successfully applied to obtain scalable distributed algorithms for the simpler class of convex optimization problems (without discrete variables), but is not yet well understood for mixed-integer variables.

This project will focus on the class of mixed integer convex programming (MICP) problems, which entail minimizing a convex function over mixed integer points in a rational polyhedron. The primary objective of this project is to extend methods for distributed convex programming to the much more challenging and much more widely applicable setting of distributed mixed-integer convex programming. The project will be based around investigating, theoretically and/or computationally, a promising approach (based on augmented Lagrangians) to these problems. Depending on the interests and strengths of the student, it could either focus on theoretical analysis, computational investigations, or tailoring the approach to give state-of-the-art algorithms in a suitable application domain.

## Project aims

### *Define the aims of the project*

The primary aim of the research project is to understand and develop theoretical, algorithmic and computational insights for the aforementioned research problem. From a theoretical point of view, the proposed research aims to resolve theoretical questions related to the correctness of algorithms for mixed integer convex programming based on the augmented Lagrangian approach. From an algorithmic perspective, the project seeks to devise methodologies to solve the MICP problems in a distributed manner whilst also addressing the questions related to convergence of these algorithms. It should be noted that while (some of) the theoretical questions pertaining to mixed integer linear programming and mixed integer quadratic programming (special subclasses of MICP) have been resolved, the algorithmic issues pertaining to even these special subclasses haven't been addressed at all.

A special case where the easy constraints are separable, leads us to consider the alternating direction method of multipliers (ADMM) and relative update schemes, which are proposed to solve convex problems separably. The project further aims to develop separable exact algorithms utilizing the strong duality results. A potential direction of further research would be to develop algorithms for general non-convex (mixed-)integer programs.

## Expected outcomes

### *Highlight the expected outcomes of the project*

The most likely outcome of the project would be journal publications in the top journals in mathematical optimization, such as the SIAM Journal on Optimization, Mathematical Programming, or Mathematics of Operations Research. Both supervisors regularly publish in these venues.

Depending on the skills and interests of the students, the project may also result in the public release of code for mixed integer convex optimization that incorporates the theoretical research to give more scalable and faster algorithms for appropriately structured problem instances.

## How will the project address the Goals of the above Themes?

*Describe how the project will address the goals of one or more of the 6 Themes listed above.*

Optimization is at the core of traditional engineering design problems, quantitative finance and learning frameworks alike. These problems are routinely formulated as optimization problems involving a mixture of integer variables (often modeling discrete choices) and continuous variables. This project aims to develop the foundational knowledge required to scale current computational methods for these problems to even larger problems, and to extend ideas for mixed integer linear programming to the more expressive class of mixed integer convex programs. This is expected to enable the computational solution of a wider range of more complex computational engineering design problems. As such, this project addresses fundamental aspects of “advanced computational engineering”.

## Capabilities and Degrees Required

*List the ideal set of capabilities that a student should have for this project. Feel free to be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.*

### For a candidate interested in theoretical exploration

Capabilities:

- (required) mathematical background (including probability, linear algebra, and some exposure to real analysis, reasonable mathematical maturity, i.e. comfortable with proofs)

Degrees:

- Bachelors and/or Masters (B.Tech, B.S., M.S., M.Sc., M.Tech.) in fields related to mathematical sciences.

### For a candidate interested in computational/algorithmic analysis

Capabilities:

- (required) programming experience
- (desirable) mathematical background
- (desirable) exposure to large-scale scientific/high-performance computing

Degrees:

- 
- :Bachelors and/or Masters (B.Tech, B.S., M.S., M.Sc., M.Tech.) in fields related to computational sciences.

## Potential Collaborators

*Please visit the IITB website [www.iitb.ac.in](http://www.iitb.ac.in) OR Monash Website [www.monash.edu](http://www.monash.edu) to highlight some potential collaborators that would be best suited for the area of research you are intending to float.*

Potential collaborators at IITB: Vishnu Narayanan (IEOR), Ashutosh Mahajan (IEOR), Shivasubramanian Gopalakrishnan (ME)

Potential collaborators at Monash: Pierre Le Bodic (IT)