IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

| | |
|---|---|
| **Project Title:** | **Discontinuous Galerkin schemes on Hybrid Meshes.** |
| **Project Number** | **IMURA0829** |

| | | |
|---|---|---|
| **Monash Main Supervisor** (Name, Email Id, Phone) | Prof. Jerome Droniou<br>jerome.droniou@monash.edu | *Full name, Email* |
| **Monash Co-supervisor(s)** (Name, Email Id, Phone) | | |
| **Monash Head of Dept/Centre** (Name,Email) | Prof. Phil Hall,<br>sci-maths-headofschool@monash.edu | *Full name, email* |
| **Monash Department:** | School of Mathematics, Faculty of Science | |
| **Monash ADGR** (Name,Email) | Prof. Peter Betts<br>Peter.Betts@monash.edu | *Full name, email* |
| **IITB Main Supervisor** (Name, Email Id, Phone) | Prof. Shivasubramanian Gopalakrishnan<br>sgopalak@iitb.ac.in | *Full name, Email* |
| **IITB Co-supervisor(s)** (Name, Email Id, Phone) | Prof. Neela Nataraj<br>neela@math.iitb.ac.in | *Full name, Email* |
| **IITB Head of Dept** (Name, Email, Phone) | Prof. Bhalachandra Puranik<br>head.me@iitb.ac.in | *Full name, email* |
| **IITB Department**: | Department of Mechanical Engineering | |

## Research Clusters:

| | |
|---|---|
| **Highlight which of the Academy's CLUSTERS this project will address?**<br>*(Please nominate JUST **one.** For more information, see www.iitbmonash.org)* | |
| 1 | *Material Science/Engineering (including Nano, Metallurgy)* |
| 2 | *Energy, Green Chem, Chemistry, Catalysis, Reaction Eng* |
| 3 | *Math, CFD, Modelling, Manufacturing* |
| 4 | *CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control* |
| 5 | *Earth Sciences and Civil Engineering (Geo, Water, Climate)* |
| 6 | *Bio, Stem Cells, Bio Chem, Pharma, Food* |
| 7 | *Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng* |
| 8 | *HSS, Design, Management* |

## Research Themes:

| | |
|---|---|
| **Highlight which of the Academy's Theme(s) this project will address?**<br>*(Feel free to nominate more than one. For more information, see www.iitbmonash.org)* | |
| *1* | Advanced computational engineering, simulation and manufacture |
| 2 | *Infrastructure Engineering* |
| 3 | *Clean Energy* |
| 4 | *Water* |
| 5 | *Nanotechnology* |
| 6 | *Biotechnology and Stem Cell Research* |
| 7 | *Humanities and social sciences* |
| 8 | Design |

## The research problem

*Define the problem*

Most models describing natural phenomena (physical, mechanical, biological, etc.) are too complex to be explicitly solved, and numerical simulations are the best way to gain practical quantitative knowledge on their solutions. Numerical analysis has made tremendous progress in the past twenty years or so, with many new algorithm being developed to tackle specific simulation issues or goals: unstructured grids made of polyhedra with generic shapes, highly variable/contrasted coefficients, requirements of high-order accuracy, etc.

These schemes however come with a cost, and running simulations can be expensive in terms of CPU time – and thus energy consumption. In modern world, efficient management of energy has become a necessity, and we cannot develop numerical methods without considering their cost and ways to reduce it.

A typical complex engineering Computational Fluid Dynamics simulation can take 256 computing cores and run for about 200 hours. Most weather and climate simulation models can employ in excess of 1000 computing cores for few hundred hours. The energy cost of these computations is in the range 500 kWh to 5 MWh per simulation.

High-order continuous Galerkin (CG) and discontinuous Galerkin (DG) methods have enjoyed much success in obtaining the higher spatial accuracy desired in numerical simulations while resolving complex physical phenomena. The success of CG and DG methods is due to their high-order accuracy and their impressive scalability on massively parallel (multi-core) computers; local high-order methods are perfectly suited for multi-core computing because the on-processor workload is large while the communication stencil is small. Both CG and DG methods are currently able to achieve efficiencies in the terascale and possibly to the petascale ranges.

The spectral convergence characteristics of the CG and DG methods gives a significant computational benefit when a calculation of a specified error or accuracy is desired. The computational cost in terms of number of floating point operations performed by the computer to achieve a particular error for the Galerkin family of methods is significantly lower when compared other established methods like Finite Volumes and Finite Differences. The higher order schemes show promise to achieve quality simulations at fraction of costs and hence much lower wall clock time to solutions as well.

## Project aims

*Define the aims of the project*

We aim to analyse and improve the energy cost of some modern high-order numerical methods designed for diffusion models on generic, unstructured grids. In particular, we will focus on Hybrid High-Order (HHO) schemes and some related methods (such as Virtual Element Methods, Hybridisable Discontinuous Galerkin, etc.). HHO schemes, and other hybrid methods, are based on face- and cell-polynomial unknowns, the later being locally eliminated via static condensation. These schemes are easy to parallelise, with minimal communication between the cores. They do however require many local computation, whose cost has to be ascertained.

We will analyse this cost, both for toy problems and more essential models (such as Navier-Stokes equations, flows in porous media, etc.), and design ways to reduce it.

The complicated geometries in most industrial CFD problems necessitates the use of complex computational meshes. These types of meshes have traditionally not been explored in the Discontinuous Galerkin framework. We will employ hybrid meshes using hexahedral, tetrahedral and transitional elements such as pyramids and prisms to solve problems on sufficiently complex geometries. Solution refinement techniques such polynomial adaptivity (p-refinement) will also be implemented.

## Expected outcomes

*Highlight the expected outcomes of the project*

- Map the energy cost for hybrid methods.
- Efficient ways to implement high-order hybrid schemes, both for local and global computations
- Software Framework for higher order simulations employing hybrid meshes.

## How will the project address the Goals of the above Themes?

*Describe how the project will address the goals of one or more of the 6 Themes listed above.*

This Project falls under the research cluster of Math, CFD Modelling and the thematic area is advanced computational engineeing. It aims to advance industrial simulation capability using newer mathematical techniques in the discontinuous Galerkin family of methods. The aim of these methods is improve simulation fidelity at lower computational cost in terms of time and energy.

## Capabilities and Degrees Required

*List the ideal set of capabilities that a student should have for this project. Feel free to be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.*

- Strong background in linear algebra
- Basic knowledge of partial differential equations
- Strong background in Numerical Methods for PDEs (?)
- Strong background in C++, Python or FORTRAN

Potential Collaborators

*Please visit the IITB website www.iitb.ac.in OR Monash Website  www.monash.edu  to highlight some potential collaborators that would be best suited for the area of research you are intending to float.*

Select up to **(4)** keywords from the Academy's approved keyword list **(available at http://www.iitbmonash.org/becoming-a-research-supervisor/)** relating to this project to make it easier for the students to apply.

**Computational Fluid Dynamics and Mechanics, Maths, Computer Simulation, Modelling and Simulation**