IITB-Monash Research Academy

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Indian Institute of Technology Bombay | MONASH University

Project Title:	**Pushing the limits of hybrid high-order schemes in case of non-regular models**	
Project Number	**IMURA0860**	
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Research Clusters:

Research Themes:

Highlight which of the Academy's CLUSTERS this project will address? *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)*		**Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)*	
1	*Material Science/Engineering (including Nano, Metallurgy)*	***1***	*Advanced computational engineering, simulation and manufacture*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*	2	*Infrastructure Engineering*
3	*Math, CFD, Modelling, Manufacturing*	3	*Clean Energy*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*	4	*Water*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*	5	*Nanotechnology*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*	6	*Biotechnology and Stem Cell Research*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*	7	*Humanities and social sciences*
8	*HSS, Design, Management*	8	Design

The research problem

Define the problem

Many models of partial differential equations (PDEs) are way too complex to be analytically solved, and obtaining qualitative information on their solutions requires the use of numerical schemes. Standard methods such as finite element schemes require specific grid geometries (with triangular/tetrahedral or quadrilateral/hexahedral cells, for example), that might not be available or efficient in practical applications – where, for example, natural meshes of an heterogeneous domain might include degenerate hexahedra, non-conforming meshes (with “hanging nodes”). The search for accuracy on sometimes coarse meshes additionally drives the use of high-order methods.

Several recent techniques have been developed to design high-order schemes on generic polygonal or polyhedral meshes, see e.g. [1]. These techniques have been applied to a range of models from fluid and solid mechanics [2], or even describing biological processes. However, for all their efficiency, these methods are sometimes not well suited to – or do not display all their strengths on – particular models such as constrained optimal control problems, non-linear fluid flow models (such as Richards equations or the Stefan problem), or models with singularity. The reason for the failure of these methods to achieve high-order lies in the reduced regularity of the exact solution, or in the difficulty of designing a stable high-order methods for singular models [3].

[1] D. A. Di Pietro, A. Ern, and S. Lemaire. “An arbitrary-order and compactstencil discretization of diffusion on general meshes based on local reconstruction operators”. In: Comput. Meth. Appl. Math. 14.4 (2014), pp. 461-472. doi: 10.1515/cmam-2014-0018.

[2] D. A. Di Pietro and J. Droniou. The Hybrid High-Order Method for Polytopal Meshes: Design, Analysis, and Applications. 2019, 516p. To appear in the Springer series “Modeling, Simulation and Applications” url: https://hal.archives-ouvertes.fr/hal-02151813.

[3] J. Droniou and R. Eymard, High-order mass-lumped schemes for nonlinear degenerate elliptic equations. 28p, 2019. Submitted. url: https://arxiv.org/abs/1902.04662.

Project aims

Define the aims of the project

We aim to design and analyse hybrid high-order schemes in situations where the continuous solution lacks the full regularity to ensure optimal accuracy of the schemes. We will in particular tackle questions around:
- constrained optimal control, in which the exact control is only piecewise smooth due to the constraint becoming active in parts of the domain,
- non-linear models in which the solution can even become discontinuous, due to the degeneracy in the model (e.g. the equation is not elliptic in the entire domain),
- singular solutions arising from a non-smooth domain geometry.

These research questions will be approached using a variety of tools, ranging from suitably designed regularisation of the model, to particular choices of locally refined meshes, to enrichment of the space of discrete solutions using specifically chosen singular functions.

Expected outcomes

Highlight the expected outcomes of the project including likelihood of patents

- Understanding on how to design and utilise high-order methods in case of singular or non-regular models – as encountered in practical applications.

- Complete rigorous analysis of the proposed schemes, ensuring their robustness including in non-standard situations (breakdown of regularity).

- Numerical tests to validate the analysis.

- Publications in top quality journals.

How will the project address the Goals of the above Themes?

Describe how the project will address the goals of one or more of the 6 Themes listed above.

This project addresses the theme *Advanced computational engineering, simulation and manufacture.* The considered models (optimal control, Stefan and Richards models) and context (non-regular solutions or domains) all appear in practical engineering and scientific situations, whose complete understanding of solutions require the usage of numerical methods. By designing high-order schemes for these models, we ensure that our approximation will have a high accuracy and thus that the qualitative behaviour highlighted by our tests can be trusted, and that the schemes are thus usable in practical engineering situations. This trust will be reinforced by the rigorous mathematical analysis conducted on the designed methods.

Capabilities and Degrees Required

List the ideal set of capabilities that a student should have for this project. Be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.

- **Master in Mathematics**, with focus on numerical analysis.
- Basic theory of elliptic and parabolic partial differential equations
- Strengths in linear algebra, functional analysis and Sobolev spaces
- Knowledge of classical Finite Element Methods
- Good coding skills, and knowledge of programming langages, with capacity to learn new ones if not already mastered (MATLAB, C++, etc.)

Potential Collaborators

Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.

Prof. Amiya K Pani, IIT Bombay
Prof. Harsha Hutridurga, IIT Bombay
Prof. Santiago Badia, Monash university
A/Prof. Ricardo Ruiz-Baier, Monash University (to arrive in January 2020).

Select up to **(4)** keywords from the Academy's approved keyword list **(available at http://www.iitbmonash.org/becoming-a-research-supervisor/)** relating to this project to make it easier for the students to apply.

Computational Fluid Dynamics and Mechanics
Maths
Computer Simulation