IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

Project Title:	Exact augmented Lagrangian duality for mixed integer convex optimization	
Project Number	**IMURA0870**	
Monash Main Supervisor (Name, Email, Phone)	James Saunderson, james.saunderson@monash.edu	*Full name, Email*
Monash Co-supervisor(s) (Name, Email, Phone)		
Monash Head of Dept/Centre (Name, Email)	Tom Drummond, tom.drummond@monash.edu	*Full name, email*
Monash Department:	Electrical and Computer Systems Engineering	
Monash ADGR (Name, Email)	Emanuele Viterbo, Emanuele.Viterbo@monash.edu	*Full name, email*
IITB Main Supervisor (Name, Email, Phone)	Avinash Bhardwaj, abhardwaj@iitb.ac.in	*Full name, Email*
IITB Co-supervisor(s) (Name, Email, Phone)		*Full name, Email*
IITB Head of Dept (Name, Email, Phone)	Bhalchandra Puranik, puranik@iitb.ac.in	*Full name, email*
IITB Department:	Mechanical Engineering	

Research Clusters:

Research Themes:

Highlight which of the Academy's CLUSTERS this project will address? *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)*		**Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)*	
1	*Material Science/Engineering (including Nano, Metallurgy)*	***1***	*Advanced computational engineering, simulation and manufacture*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*	2	*Infrastructure Engineering*
3	*Math, CFD, Modelling, Manufacturing*	3	*Clean Energy*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*	4	*Water*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*	5	*Nanotechnology*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*	6	*Biotechnology and Stem Cell Research*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*	7	*Humanities and social sciences*
8	*HSS, Design, Management*	8	Design

The research problem

Define the problem

Mixed integer convex programming (MICP) entails minimizing a convex function over mixed integer points in a rational polyhedron. An enormous range of optimization problems in areas such as engineering design, logistics, and statistics, can be modelled as MICPs. Nevertheless, it remains computationally challenging to solve large-scale instances of these problems.

The research problem in context focuses on understanding, from a mathematical and computational point of view, the relationship between a MICP and a related optimization problem, called an augmented Lagrangian dual (ALD). An ALD augments the usual Lagrangian dual with a weighted nonlinear penalty on the dualized constraints. Often we have optimization problems where optimizing over a subset of constraints is relatively easier. By dualizing the harder constraints using an appropriate augmenting function as penalty we can solve the problem over the set of relatively easier constraints. However, doing so often leads to a duality gap between the primal MICP and the corresponding ALD.

The problem of interest is to identify and classify the augmenting functions that yield no duality gap with a finite penalty parameter and also to provide bounds on the size of the penalty parameter. Progress on this problem is expected to lead to algorithms for MICPs that can be solved in a distributed manner, and hence scaled to much more challenging problem sizes.

Project aims

Define the aims of the project

The primary aim of the research project is to understand and develop the theoretical, algorithmic and computational insights for the aforementioned research problem. In particular, the proposed research aims to discover asymptotic behaviour of the duality gap with respect to the penalty parameter of the ALD. In addition, we aim to derive necessary and sufficient conditions on the class of penalty functions to have zero duality gaps with finite penalty parameter.

A special case where the easy constraints are separable, leads us to consider the alternating direction method of multipliers (ADMM) and relative update schemes, which are proposed to solve convex problems separably. The project further aims to develop separable exact algorithms utilizing the strong duality results. A potential direction of further research would be to develop algorithms for general non-convex (mixed-)integer programs.

Expected outcomes

Highlight the expected outcomes of the project including likelihood of patents

The most likely outcome of the project would be journal publications in the top journals in mathematical optimization, such as the SIAM Journal on Optimization, Mathematical Programming, or Mathematics of Operations Research. Both supervisors regularly publish in these venues.

Depending on the skills and interests of the students, the project may also result in the public release of code for mixed integer convex optimization that incorporates the theoretical research to give more scalable and faster algorithms for appropriately structured problem instances.

How will the project address the Goals of the above Themes?

Describe how the project will address the goals of one or more of the 6 Themes listed above.

Engineering design problems are routinely formulated as optimization problems involving a mixture of integer variables (often modeling discrete choices) and continuous variables. This project aims to develop the foundational knowledge required to scale current computational methods for these problems to even larger problems, and to extend ideas for mixed integer linear programming to the more expressive class of mixed integer convex programs. This is expected to enable the computational solution of wider range of more complex computational engineering design problems. As such, this project addresses fundamental aspects of “advanced computational engineering”.

Capabilities and Degrees Required

List the ideal set of capabilities that a student should have for this project. Be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.

Capabilities:

- mathematical background (including probability, linear algebra, and some exposure to real analysis, reasonable mathematical maturity, i.e. comfortable with proofs)
- (required) programming experience
- (desirable) exposure to large-scale scientific/high-performance computing

Degrees:

- Bachelors and/or Masters (B.Tech, B.S., M.S., M.Sc., M.Tech.) in fields related to mathematical sciences.

Potential Collaborators

Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.

Potential collaborators at IITB: Vishnu Narayanan (IEOR), Ashutosh Mahajan (IEOR), Shivasubramanian Gopalakrishnan (ME)
Potential collaborators at Monash: Pierre Le Bodic (IT)

Select up to **(4)** keywords from the Academy’s approved keyword list **(available at http://www.iitbmonash.org/becoming-a-research-supervisor/)** relating to this project to make it easier for the students to apply.

Data Science, optimisation, algorithms
Maths