IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

Project Title:	**Mathematical modelling and numerical methods for the sliding of soft tissues**	
Project Number	**IMURA0992**	
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Research Clusters:

Highlight which of the Academy's CLUSTERS this project will address?
*(Please nominate JUST **one**. For more information, see www.iitbmonash.org)*

1	*Material Science/Engineering (including Nano, Metallurgy)*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*
3	*Math, CFD, Modelling, Manufacturing*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*
8	*HSS, Design, Management*

Research Themes:

Highlight which of the Academy's Theme(s) this project will address?
(Feel free to nominate more than one. For more information, see www.iitbmonash.org)

1	*Advanced computational engineering, simulation and manufacture*
2	*Infrastructure Engineering*
3	*Clean Energy*
4	*Water*
5	*Nanotechnology*
6	*Biotechnology and Stem Cell Research*
7	*Humanities and social sciences*
8	Design

The research problem

The contact and sliding of tissues occurs as a key mechanism in many physical and biological contexts. A remarkable example is diarthrodial joints, present all over the human body, and allowing a wide range of functional motion [1]. In this case, the tissues involved include articular cartilage, soft connective tissue, and bones. All of these components can be regarded as deformable porous structures interacting through lubricated friction and sustaining large compression and shear forces during normal muscle activity [2]. Soft tissues are naturally hydrated, consisting of a solid phase and an interstitial fluid phase. In the case of highly deformable biological tissues, the porous skeleton may undergo large strains, and the fluid may also play a significant role in maintaining the large strain capability of the material. The limitations of the classical theory of poroelasticity based on infinitesimal deformation kinematics become apparent in the mechanics of soft hydrated tissues subjected to large strains, and one needs to resort to the porohyperelastic regime.

Experimental investigation of failure under extreme conditions has obvious limitations, and mathematical models and computational methods can help revealing fundamental mechanisms occurring at different spatio-temporal scales, which are currently out of reach for in-vivo studies. This is where mathematical modelling and numerical simulation can certainly help to understand and predict the behaviour of the underlying systems in more detail. However, due to the inherent complexity of the coupling structures and nonlinearity of the involved equations, the rigorous analysis of the resulting models and the design of numerical methods suitable for their computational simulation, remain very far from trivial. For this purpose, we will construct accurate, robust and reliable mixed finite element methods for the discretization of the underlying equations. These methods enjoy many features typically sought in discretisations for coupled problems involving flow, deformation, and transport phenomena. For instance, local satisfaction of divergence-free phase velocities that will prevent artificial mass sinks or sources. In addition, we are interested in computing additional fields directly (such as volume fractions, or stresses), without resorting to numerical postprocessing [7]. Another important aspect is the construction of robust and efficient solvers [5]. Many open questions remain from the physical, mathematical, and numerical viewpoint; and the theoretical models and methods emanating from this project will advance the state-of-the-art in investigating a range of applications in biomechanics, including cartilage-on-cartilage interactions that are of importance, for instance, in walking and shock response [2]; large-deformation and strain-dependent permeability in common joint loadings [1]; mass transport and drug delivery through arterial walls under the presence of pulsating flow [3]; in the formation of myocardial edema [4,6]; and modifications in the progressive compaction of collagen layers in the eye, which lead to the formation of glaucoma.

The present study aims to investigate the contact mechanics of the porohyperelastic materials using semi-analytical techniques and designing/testing novel numerical tools based on physics-preserving discretisations with mixed finite element methods.

References

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2. Ateshian GA, Maas S, Weiss JA. *Finite element algorithm for frictionless contact of porous permeable media under finite deformation and sliding*. Journal of Biomechanical Engineering. 2010;132(6).
3. Zakerzadeh R, Zunino P. *A computational framework for fluid–porous structure interaction with large structural deformation*. Meccanica. 2019;54(1):101–121.
4. Freitas Reis R, Weber Dos Santos R, Martins Rocha B, Lobosco M. *On the mathematical modeling of inflammatory edema formation*, Computers and Mathematics with Applications, 78 (2019), pp. 2994–3006.
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7. Farrell PE, Gatica LF, Lamichhane BP, Oyarzua R, Ruiz-Baier R. *Mixed Kirchhoff stress - displacement - pressure formulations for incompressible hyperelasticity*, Computer Methods in Applied Mechanics and Engineering, 374 (2021), p. e113562.

Project aims

1) Construct suitable porohydromechanical models for the contact dynamics of soft tissue. Using asymptotic analysis, study the properties of the coupled multiphysics problems in different contexts, and state the main research questions coming from (at least one of) the applicative problems.
2) Design and implement mixed finite element formulations for coupled large-strain poromechanics. This step involves getting acquainted with advanced aspects of numerical techniques and programming languages such as Python and C++.
3) Perform the numerical analysis (stability, solvability, convergence) of the mixed schemes, addressing also more theoretical questions.
4) Use the tools developed in points 1-2 to investigate models for: a) cartilage-on-cartilage interactions that are of importance, for instance, in walking and shock response; b) large-deformation and strain-dependent permeability in common joint loadings; c) mass transport and drug delivery through arterial walls under the presence of pulsating flow; and d) modifications in the progressive compaction of collagen layers in the eye, which lead to the formation of glaucoma.

Expected outcomes

The overall mechanical and computational findings will improve the current understanding of sliding of soft tissues in diverse applicative contexts. The main outcomes of the project are

- A novel framework for the theoretical modelling and numerical analysis of a class of coupled systems of PDEs arising from continuum mechanics
- Robust and efficient discretisations will be advanced and their capabilities will be fully tested
- Results will be validated against benchmark data and experimental observations
- Computational codes and preprints of our scientific reports will be made publicly available in online repositories

How will the project address the Goals of the above Themes?

Specific applications of the formalisms outlined above include growth and proliferation of embryonic cells, macroscopic interaction between cardiac muscle deformation and perfusion, articular cartilage characterization, design and testing of smart materials. The outcomes of this project will contribute to fill the gap between advanced techniques currently employed by practitioners and the sound mathematical foundation of novel models and numerical methods. Being a highly visible subject, it is expected that the project will attract several collaborations from which the IITB-Monash Research Academy will benefit. We also foresee the possibility of building new engagements with the biomedical industrial sector. All these items have a clear relevance to the goal “Advanced computational engineering, simulation and manufacture”.

Capabilities and Degrees Required

List the ideal set of capabilities that a student should have for this project. Feel free to be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.

Degree: MSc in Mathematics, MSc in Engineering, or MSc in Computer Science

Capabilities:
Continuum mechanics
Partial differential equations
Numerical methods
Scientific computing

Potential Collaborators

Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.

Neela Nataraj, IITB Mathematics. She knows the overall topic very well and has participated already in several projects from the IITB-Monash academy.

Mark Flegg, Monash Mathematics. He is a specialist on one applicative aspect of this project.